Finding the Mass of Sagittarius A* by Measuring the Orbit of S2 *

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Problem: From data describing the orbit of the star S2 around the black hole at the centre of the Galaxy, Sagittarius A*, calculate the black hole's mass in solar mass units. The period (P) of S2 is 15.559 years; the orbital eccentricity (e) is 0.88; and the parigalacticon distance (d) is $1.835 * 10^{13}$ m. (Data are from Eisenhauer et al. (2003) and Schödel et al. (2002).)

Hints:

Use Kepler's third law and the equation of an ellipse.
Simplify calculations by using Solar System units (e.g. AU and solar mass).

Data:

$e := 0.88$

$$0.88 \quad \textbf{(1)}$$

$d := 1.835 \cdot 10^{13}\,\mathrm{m}$

$$1.835000000\ 10^{13}\ \mathrm{m} \quad \textbf{(2)}$$

$P := 15.559 \cdot 365.25 \cdot 24 \cdot 60 \cdot 60\,\mathrm{s}$

$$4.910046984\ 10^{8}\ \mathrm{s} \quad \textbf{(3)}$$

$G := 6.67259 \cdot 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$ # *gravitational constant*

$$\frac{6.672590000\ 10^{-11}\ \mathrm{m^3}}{\mathrm{kg\ s^2}} \quad \textbf{(4)}$$

$sm := 1.9891 \cdot 10^{30}\,\mathrm{kg}$ # *solar mass*

$$1.989100000\ 10^{30}\ \mathrm{kg} \quad \textbf{(5)}$$

Useful Equations:

d = a*(1-e): # ellipse

$M = 4 * \pi^2 * a^3/G * P^2$: # *Kepler's third law*

Solution:

From the equation for the ellipse, determine the semi-major axis of S2:

$solve(d = a * (1 - e), a)$

$$1.529166667\ 10^{14}\ \mathrm{m} \quad \textbf{(6)}$$

$a := 1.529166667\ 10^{14}\ \mathrm{m}$

$$1.529166667\ 10^{14}\ \mathrm{m} \quad \textbf{(7)}$$

From Kepler's third law:

$$evalf\left(M = \frac{\left(4 * \pi^2 * a^3\right)}{G \cdot P^2}\right)$$

$$M = 8.775229202\ 10^{36}\ \mathrm{kg} \quad \textbf{(8)}$$

In solar mass units:

$$\frac{\left(8.775229202\ 10^{36}\ \mathrm{kg}\right)}{sm}$$

$$4.411658138\ 10^{6} \quad \textbf{(9)}$$

The mass of Sagittarius A* is approximately four million times the mass of the Sun. (The currently accepted value, as reported in the text, is $3.7 \pm 1.5 * 10^6$.)

Reference

Eisenhauer, F., Schödel, R., Genzel, R., Ott, T., Tecza, M. and Abuter, R. (2003). A Geometric Determination of the Distance to the Galactic Center. *The Astrophysical Journal Letters*, **597**, 2, L121-L124.

Schödel, R. et al. (2002). Closest Star Seen Orbiting the Supermassive Black Hole at the Center of the Milky Way. *Nature*, **419,** 694-696.